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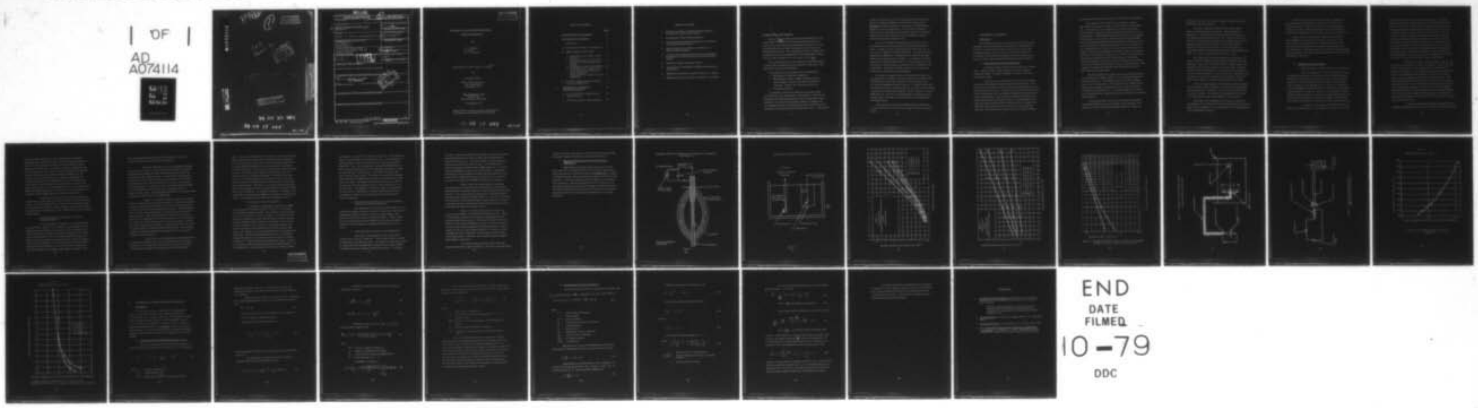

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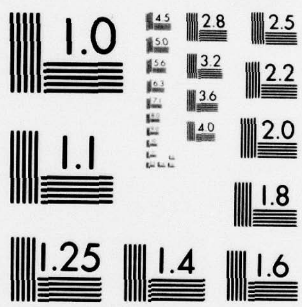
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RESEARCH ON MAGNETOHYDRODYNAMIC SOUND TRANSDUCERS

BY

E. J. Hellund
J. T. Naff
R. C. Brumfield

Final Report on Contract Nonr - 3117(00) new

TO

Acoustics Branch

Office of Naval Research
Navy Department
Washington, D. C.

MHD RESEARCH, INC.
PO Box 1815
Newport Beach, California

31 December 1961

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INTRODUCTION AND SUMMARY

Contract Nonr-3117(00) indicates that MHD Research, Inc. "shall conduct such theoretical and experimental studies as to provide basic information on the generation of sound waves in an electrolyte (such as sea water) by high current pulsing of submerged induction coils. The study shall provide, if possible, the modeling criteria for the applicability of the phenomena to underwater acoustic transducers for use in sonar work."

The contract effort was concentrated in the periods 1 April to 30 November 1960 (Phase I) and 1 March to 31 December 1961 (Phase II), an activity of 18 months.

The program can be described in three related parts:

a. Development of laboratory apparatus.

b. Experimental studies of MHD sound generation by electromagnetic induction and direct conduction in salt water, sulfuric acid solutions and mercury.

c. Theoretical studies.

Development of laboratory apparatus proceeded rapidly and effectively in the early phases of the program. The first experiments with submerged induction coils in simulated sea water resulted in very weak acoustic signal outputs even though coils were pulsed to levels of 100,000 amperes and in several cases to destruction by mechanical and electromagnetic forces. The acoustic

output from fluid body forces was dominated by electrochemical agitation of metal components. The electrical output of the detecting hydrophone tended to be submerged in ambient electromagnetic noise in the laboratory. The low level of the fluid body forces was obviously due to the high resistivity of the (simulated) sea water and the low level of the circulating currents induced by pulsing the magnetic field.

In order to eliminate the dependence on induced current, experiments were devised to pass current directly through sea water. A large increase in sound output was observed, due, it was found, to impulsive heating of the water. This thermal effect dominated the MHD effect, although both are inherent and inseparable in the conduction type transducer. A transducer was then designed to isolate the thermal effect. It was found to work very effectively at the high frequency and power levels employed in the experiments. This type of transducer appears as a promising prospect for further study and development.

The theory of magnetoacoustics and thermal acoustics was developed by E. J. Hellund. The basic differential equations were derived and the form of the integrals for the pressures in the far-field of homogeneous media are given. It should be noted that the dominant role of the thermal effects in sea water was discovered in the last few months of the program. Earlier efforts to evaluate MHD effects in sea water were handicapped by interference from ambient noise, vibration of metal parts and by sound generated by thermal effects.

The discovery of the role of thermal effects is believed to be novel and to have practical value at higher frequency and power levels.

I EXPERIMENTAL PROGRAM

Introduction

The experimental program included the construction of the equipment and the performance of experiments to determine the feasibility of production of sound waves directly in electrically conducting fluids by means of electric and magnetic fields. A description of the experimental apparatus will first be given and then an account of the experiments and results will be reported.

A. Experimental Facilities and Apparatus

The general experimental facility consists of a testing tank, a capacitor bank, a high voltage power supply, a pulse triggering console, recording equipment, and the various transducers under study.

The tank is a five foot by four foot by four foot wooden tank lined with fiberglass-reinforced plastic on the interior. The tank is used to contain the solution (sodium chloride solutions, etc.) in which the tests were run. The tank was often filled with water and the test solution placed in a partially submerged-acoustically transparent, open-top bag. This facilitated changing the solutions and reduced the volume of solution necessary. The dimensions of the tank were chosen such that with a frequency of 5 kc or higher, the first full cycle could be received before the arrival of the first reflection. This allowed true measurements of the peak pressure

from the pulsed sources used without interference from reflections.

The capacitor bank consists of four 29.4 mf, 14.4 KV, Type NRG low inductance storage capacitors built by Cornell-Dubilier. Each capacitor was equipped with a separate spark gap switch similar in operation to the conventional "trigatron" in a co-axial geometry. Individual spark gaps were used for each condenser because the peak current of the entire bank discharging simultaneously through one spark gap would be sufficient to erode the switch electrodes enough to make them unreliable after a few firings. In the arrangement used, the electrodes can easily withstand several hundred firings before polishing is necessary. Also, with separate spark gaps, any number of capacitors can be used at any given time allowing great flexibility in total energy storage. An argon or nitrogen atmosphere is used in the switches to eliminate formation of corrosive compounds commonly found on air gap electrodes. The completely-enclosed pressurized design of the spark gap switch also results in quiet operation.

The high voltage power supply used to charge the capacitors was obtained on the surplus market. It is an RA-38 manufactured by Western Electric. The supply delivered 0-15 KV at 0-500ma. from a 110 V, 60 cps source. The capacity of this supply allowed charging the capacitors rapidly which was convenient as well as favorable to good capacitor life. The power supply was provided with conventional safety equipment to provide protection of personnel and equipment.

An additional safety feature which consisted of automatic shorting mechanisms for each capacitor in the main bank. The capacitors remain shorted until the high voltage was turned on.

When the bank was fired, the power supply automatically turned off and the capacitors shorted.

The pulse triggering console provided a high voltage pulse for initiating the main discharge. The discharge was triggered by a 15 KV pulse formed by discharing a 0.02 mf., 15 KV capacitor through a thyratron circuit. A separate 0.02 mf. capacitor is provided for each gap, but all trigger capacitors discharge through the same thyratron. This synchronizes the discharge of all the main capacitors. In one experiment, the triggering console was used alone to power the experimental transducer. This work is described under Section 1 B.2.

The connection from the spark gap to the transducer was made by using four RG8/U co-axial cables in parallel from each spark gap. The sixteen resulting leads were cabled together and brought out of the capacitor bank to a collector ring. From the collector ring to the transducer, a large rigid co-axial conductor was used. This resulted in a very low inductance connection from the capacitor bank to the transducer that is flexible to allow rotation of the transducer for sound distribution measurements.

The receiving hydrophone was an Atlantic Research Model LC-32. The flat region of frequency response of this hydrophone was from 0.05 cps to 100 Kcps at a level of 101 db referenced to 1 volt per microbar. The output of the hydrophone was fed into a very high input impedance pre-amplifier with a gain of 1.

Recording was accomplished by feeding the output of the pre-amplifier into a Tektronic Type 545 oscilloscope with a Type-D plug-in vertical amplifier. The oscilloscope trace is photographed with a Land Polaroid camera.

The horizontal sweep of the oscilloscope is triggered at the same time as the thyratron is triggered. Therefore, the sound output pulse is delayed by an amount corresponding to the transit time of the sound from the projector to the hydrophone. This time corresponded to the calculated transit time in all cases.

In the experiments with direct conduction transducers, the hydrophone was separated from the salt solution by a thin plastic bag containing fresh water. This proved necessary for protection of the hydrophone from the high voltage appearing in the salt solution. The danger results from the electrically grounded noise shield of the hydrophone and the proximity of the projector and hydrophone in the tank.

B. MHD Transducer Experiments

1. Sound Generation in Sodium Chloride-water Solutions

The first experiments on sound generation by means of magnetohydrodynamic interactions was by the induction method. The principle of operation involved immersing a coil (an inductive element) in a conducting medium (Na Cl solution) and then pass a large oscillating current through the coil. This was accomplished by the underdamped discharges of the previously-described capacitor bank connected to the inductive element. The resulting oscillating magnetic field then induced currents in the conducting medium, and the interaction of these currents with the magnetic field produced body forces on the medium. The body forces generated sound waves.

A transducer was constructed to determine the magnitude of the sound output. The transducer consisted of a six-turn

flat helix wound of one-inch wide conducting strip. The inside diameter of the coil was three inches and the outside diameter five inches. The coil was initially potted in a machinable epoxy resin. The epoxy proved too brittle under the magnetic stress of the coil in operation and electrical breakdown occurred after fracture of the epoxy. An epoxy loaded with asbestos fibers proved to be of sufficient strength to withstand the magnetic stresses. The coil was housed in a brass housing with a lucite cover and heavy brass back plate to isolate it from the conducting medium. The coil was provided with an air space between the housing and the medium in an attempt to reduce mechanical coupling. In this configuration, the peak magnetic field was calculated to be 35 kilogauss occurring at a 14 KC rate with a total energy storage of 6000 joules.

When the transducer was submerged in the salt solution and the coil energized, a large sound signal was observed. The salt (Na Cl) solution was a saturated solution of 0.22 mho-cm^{-1} electrical conductivity. It was immediately noted that the frequency of the received sound was not double the frequency of the under-damped capacitor-inductance discharge as theory indicates. The sound was subsequently found to be due to vibrations of the housing containing the coil. A lucite back plate was constructed for the coil and the experiment repeated. The results were similar although the pressure peak of the sound did drop by a factor of approximately four. This sound was subsequently (by operating the transducer in distilled water) found to be due to vibratons of the coil itself coupling to the mounting structure.

In order to remedy this situation an experiment was designed to mechanically decouple the coil from the conducting medium.

The experiment consisted of a tube containing the salt solution suspended from a special stand and acoustically insulated from the floor. The coil was placed around the tube, but physical contact between the coil and tube was avoided. With this arrangement, the coil was energized. There was a weak but perceptible sound signal observed of the proper frequency for sound of MHD origin. No accurate data was collected since no calibrated hydrophone was available at the time of the experiments and reverberations in the tube would have prevented precise measurements. However, the signals observed were only 10 db above the ambient noise of the hydrophone and amplification system. Therefore, it was concluded that producing sound by MHD induction is an inefficient process.

Calculations showed that, due to the low electrical conductivity of the salt solution, relatively small currents were being induced. It was determined that if currents were produced by means of electrodes in the medium, that the coupling could be made stronger due to the higher current levels. This conclusion led to the next series of experiments.

2. Sound Generation in NaCl Solution by a Direct Conduction Method

To test this plan, a transducer of co-axial geometry was constructed. Figure 1 is a schematic of this transducer and a diagnammatic explanation of its operation. This configuration was chosen because it is relatively free of electromagnetic forces generated in the metal parts. The current passed down the center conductor of the co-axial geometry creating an axi-symmetric magnetic field around the transducer. Then the current leaves an electrode at the bottom of the transducer and returns to an electrode at the top of the transducer traversing at right angles the magnetic field

lines while passing through the conducting medium. This gives rise to a body force outward from the transducer.

When the experiment was at first attempted the large capacitor bank was used. A large signal was immediately apparent. A smaller power supply was then used, namely the nine joule triggering unit for the condenser banks. A considerable sound pressure was again received. Since this source was mounted in the tank and the signals were free from reverberations, accurate measurements could be performed. At this time, also, the Atlantic Research Model LC-32 hydrophone was available so that calibrated sound level studies could be made. The following experiments were then performed to determine if the sound was of MHD origin.

Figure 2 is a general schematic outline of the experimental arangement used to test this transducer. Results were plotted in Figure 3 which shows sound pressure versus total energy storage at various distances from the projector transducer. At total energies above 4 joules, the sound pressure is a linear function of the total energy storage. Under these conditions of operation the current came out as a single pulse of total time duration of 3.5 microseconds. The capacitance of the storage system is 0.08 microfarad. Figure 4 is a replot of the test data of Figure 3 showing sound pressure versus reciprocal distance for different total energy storage. The resulting straight lines indicate that the pressure falls off as the reciprocal distance from the projector.

At this time, it was realized that a contribution could be made to the sound pressure due to impulsive thermal expansion of the water. A theoretical calculation was carried out which indicated that the sound pressure was primarily due to the thermal

effect. The results obtained for this transducer were then compared with the theoretical determination of thermal pressure pulsations. It was found that the sound pressure should be directly proportional to the energy and should fall off as the reciprocal of the distance from the transducer. These are the results actually found in Figures 3 and 4. A calculation of the absolute peak pressure due to the thermal effect was completed and Figure 5 resulted. Within the accuracy of the determination of the parameters used in the calculation, the theory of thermal pressure pulses and the experimental measurements agree. Therefore, it was determined that the transducer was producing primarily thermal pulsations and that the MHD effects were secondary. To confirm this further, a transducer was assembled that minimized the MHD effect and optimized the thermal effect. This experiment is described in the following section.

3. Confirmation of the Thermal Effect

A transducer was designed to minimize the MHD effect and optimize the thermal effect. Figure 6 is a schematic of the thermal transducer system. The transducer itself consisted of a concave back plate with a 75 cm radius. A curved screen was placed one centimeter in front of the plate and current was passed through the NaCl solution (conductivity 0.04 mho-cm^{-1}) from the screen to the plate. The screen was very poorly coupled to the water acoustically so that vibrations of the screen produce little sound. The magnetic forces on the medium itself are minimized by this design. However, if thermal pulsations occur, they should transmit through the screen and be focused at a point 75 cm from the projector. With only 375 joules total energy storage, a signal of the proper frequency (double the capacitor ringing frequency, as in the MHD effect, and as predicted by theory) at a level of 120 db

referenced to 1 $dyne/cm^2$ was observed at the focus. The transducer showed a gain of 30 db in the forward direction. The conclusion of this experiment is that the thermal effect is large and masks the MHD effect. The ratio of the pressure produced by MHD effects to that produced by thermal effects was calculated and found to be 10τ for 0.04 $mho\text{-}cm^{-1}$ solutions (e.g., sea water), where τ is the pulse duration of the current. From this it is seen that the thermal effect dominates the MHD effect except at low frequency where both effects are small. This leads to the conclusion that even in the induction case, the small signals observed were probably due to the thermal effect. The MHD effect requires a larger conductivity (by several orders of magnitude) to be larger than the thermal effect providing the coefficient of thermal expansion and specific heat remain comparable.

4. Sound Generation in Sulfuric-Acid-Water Solutions by Induction and Direct Conduction

The experiments in Section B.1 and B.2 above were conducted in H_2SO_4 solutions of electrical conductivity 0.74 $mho\text{-}cm^{-1}$. Although the measured pressure levels are slightly higher, the general result was the same as with NaCl solutions. Theory shows that the thermal effect masks the MHD effect and extensive experimentation was not completed on H_2SO_4 solutions because of this factor.

5. Sound Generation in Mercury by the MHD Effect

Mercury at room temperature has an electrical conductivity of 1.04×10^4 $mho\text{-}cm^{-1}$. This is more than five orders of magnitude larger than sea water. This makes it an attractive liquid for sound generation by MHD effects because of the strong coupling available between the magnetic field and the mercury. To test the

mercury, a cell was assembled that contained one pound of mercury and fitted directly on the same mechanically decoupled tube as described in Section B. 1. The same magnetic coil and capacitor system was used as in Section B. 1 also. A very high level of sound was emitted from the mercury MHD transducer. An uncalibrated hydrophone was used to record the signals. Calibration against the LC-32 provided the estimate that the sound levels were on the order of 100 db referenced to 1 dyne-cm^{-2} at 1 yard from the source at 14 KC and with 6000 joules total energy storage.

Figure 7 shows the experimental arrangement. Reverberations prevented accurate measurements, but by taking peak pressures at constant frequency, relative measurements could be made. It was found that the peak pressure was proportional to the total energy storage as shown in Figure 8. This is in accordance with the theory as the energy storage is proportional to the square of the peak current, and the theory shows that peak pressure is proportional to the square of the peak current.

Figure 9 is a plot of peak pressure output as a function of the distance of the coil from the mercury cell. This is to some extent a measure of the degree of coupling between the magnetic field and the mercury. The field on the axis of the coil falls off as the reciprocal of distance cubed. A plot of reciprocal distance cubed times an appropriate constant necessary to superimpose the curves is also shown in Figure 9. As can be seen the coupling falls off faster than R^{-3} at low outputs but follows quite well at high outputs. Therefore, this indicates strong coupling at high magnetic field values.

The sound pressures would be higher than that indicated (100 db) for higher frequencies. Because of the exploratory

nature of the tests with mercury, no care was taken in optimizing the acoustic match of the mercury cell to the water tube.

C. Summary of Experimental Sound Generation by MHD Effects

Experiments were conducted on fluids with low (less than 1 mho-cm^{-1}) and high (10^4 mho-cm^{-1}) electrical conductivity. In the low electrical conductivity solutions the MHD effect is masked by thermal pressure pulsations and does not appear useful for sound generation. The high conductivity fluids do appear to be usable for sound generation. The main problem in the use of mercury is its extremely high density and acoustic impedance mismatch with water. Other high-conductivity fluids with lower densities such as sodium-potassium alloys do exist and appear promising for use in sound generators.

Schematic of Physical Configuration and Operation of Conduction-Type Projector

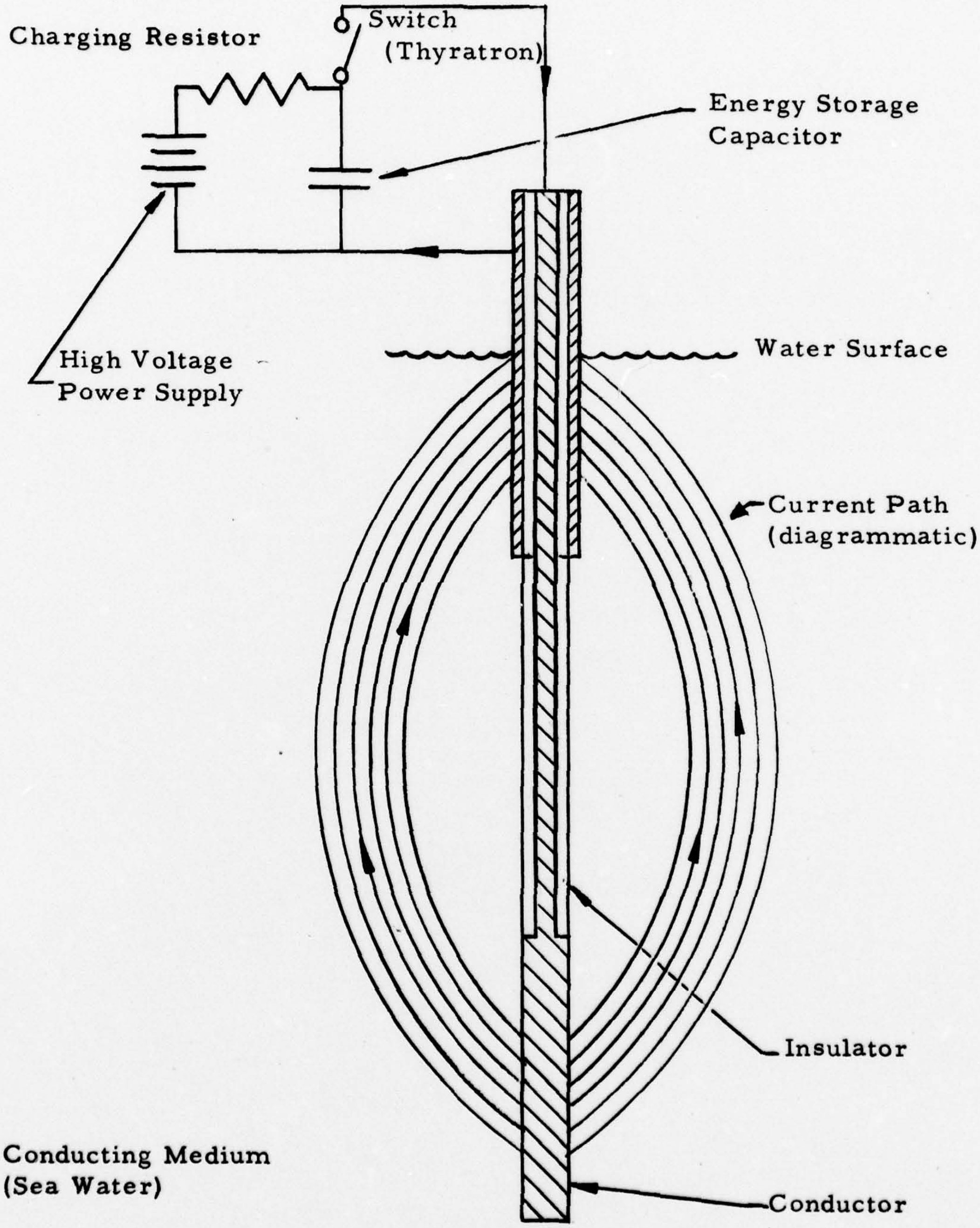


Figure 1

Arrangement of Experimental Apparatus

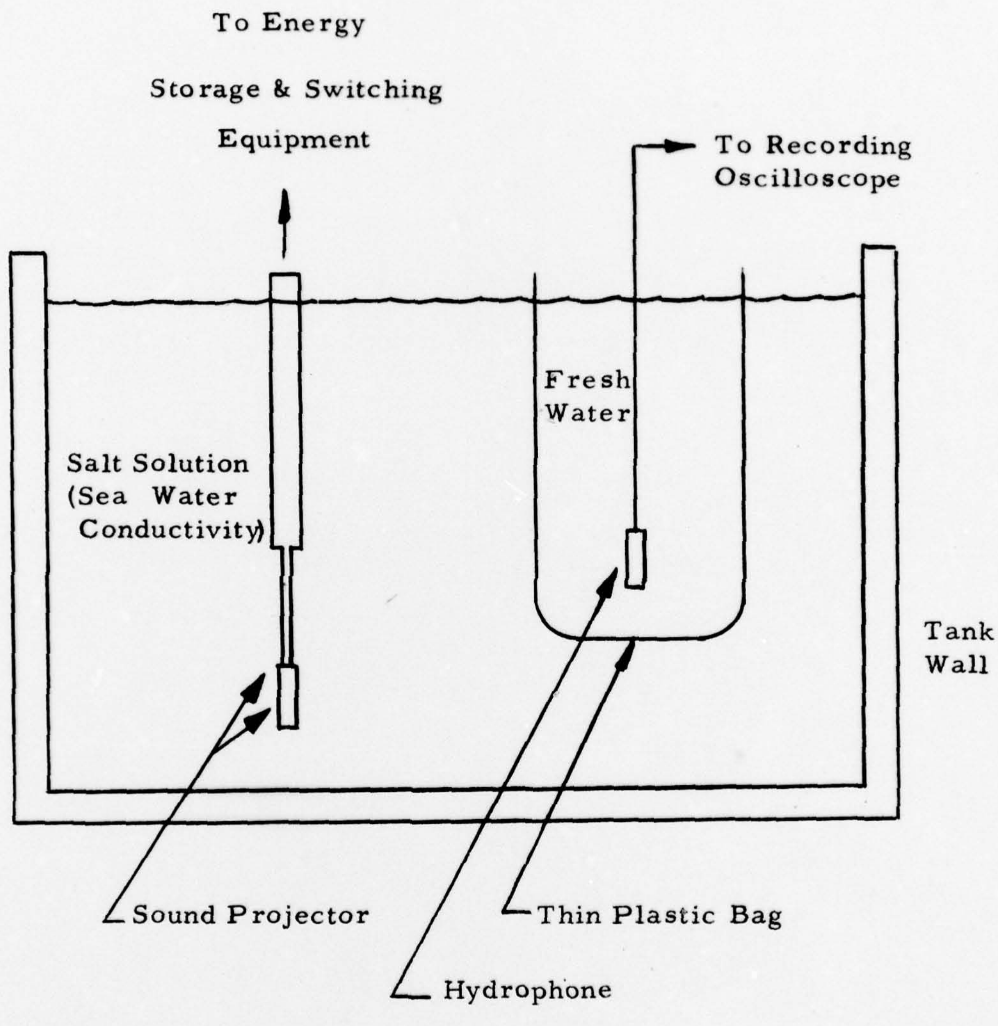


Figure 2

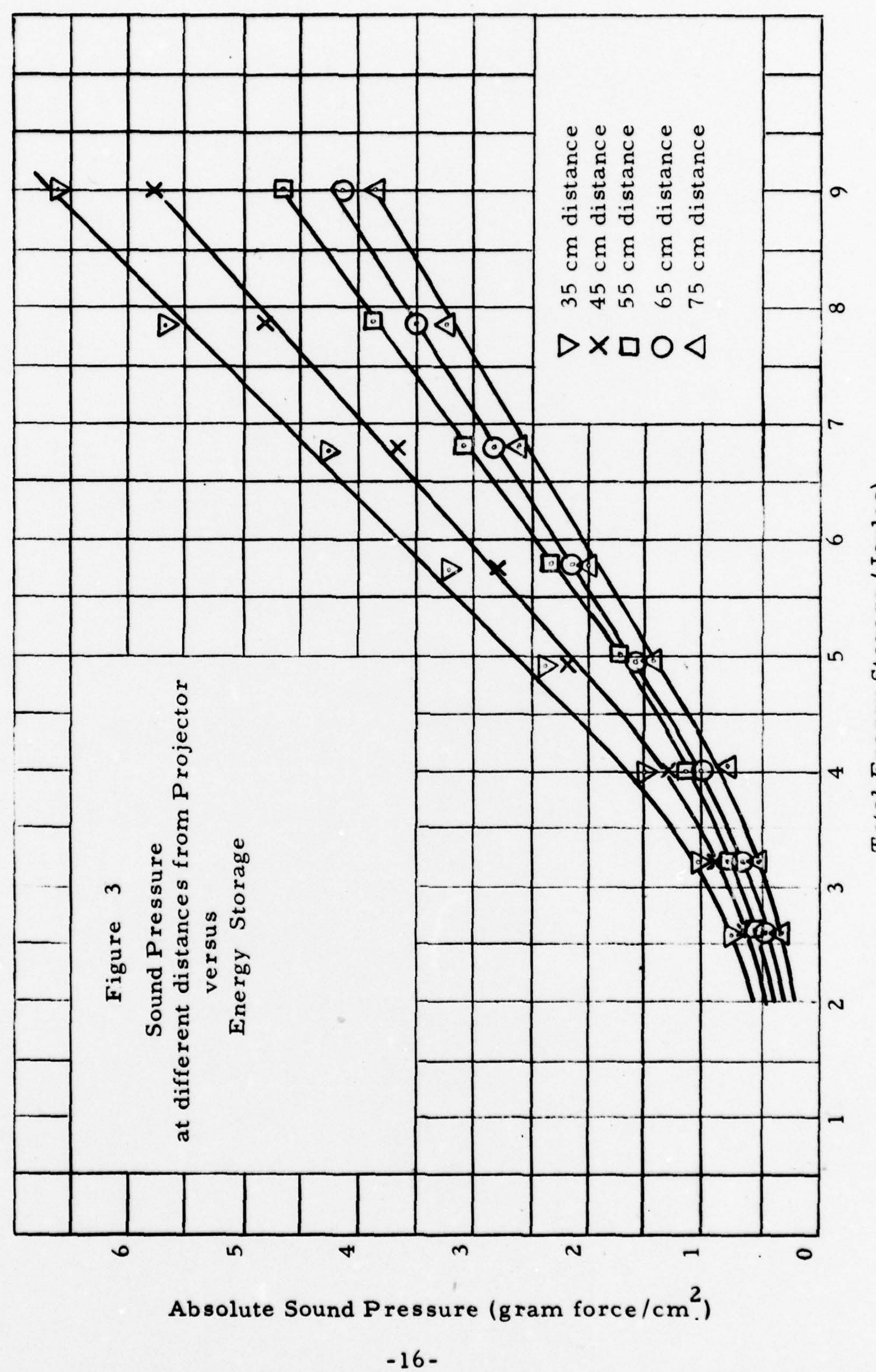


Figure 3
Sound Pressure
at different distances from Projector
versus
Energy Storage

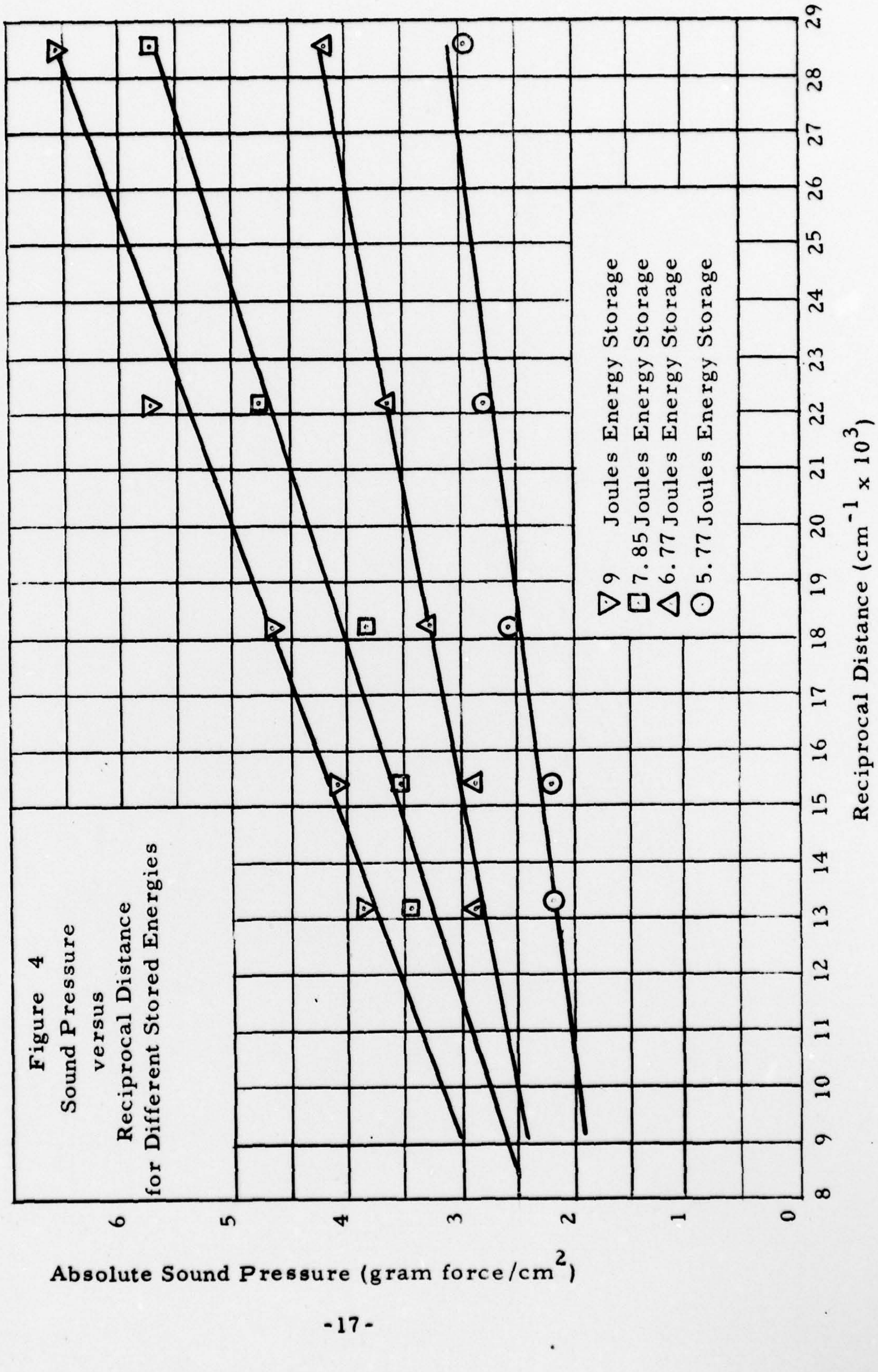


Figure 4
Sound Pressure
versus
Reciprocal Distance
for Different Stored Energies

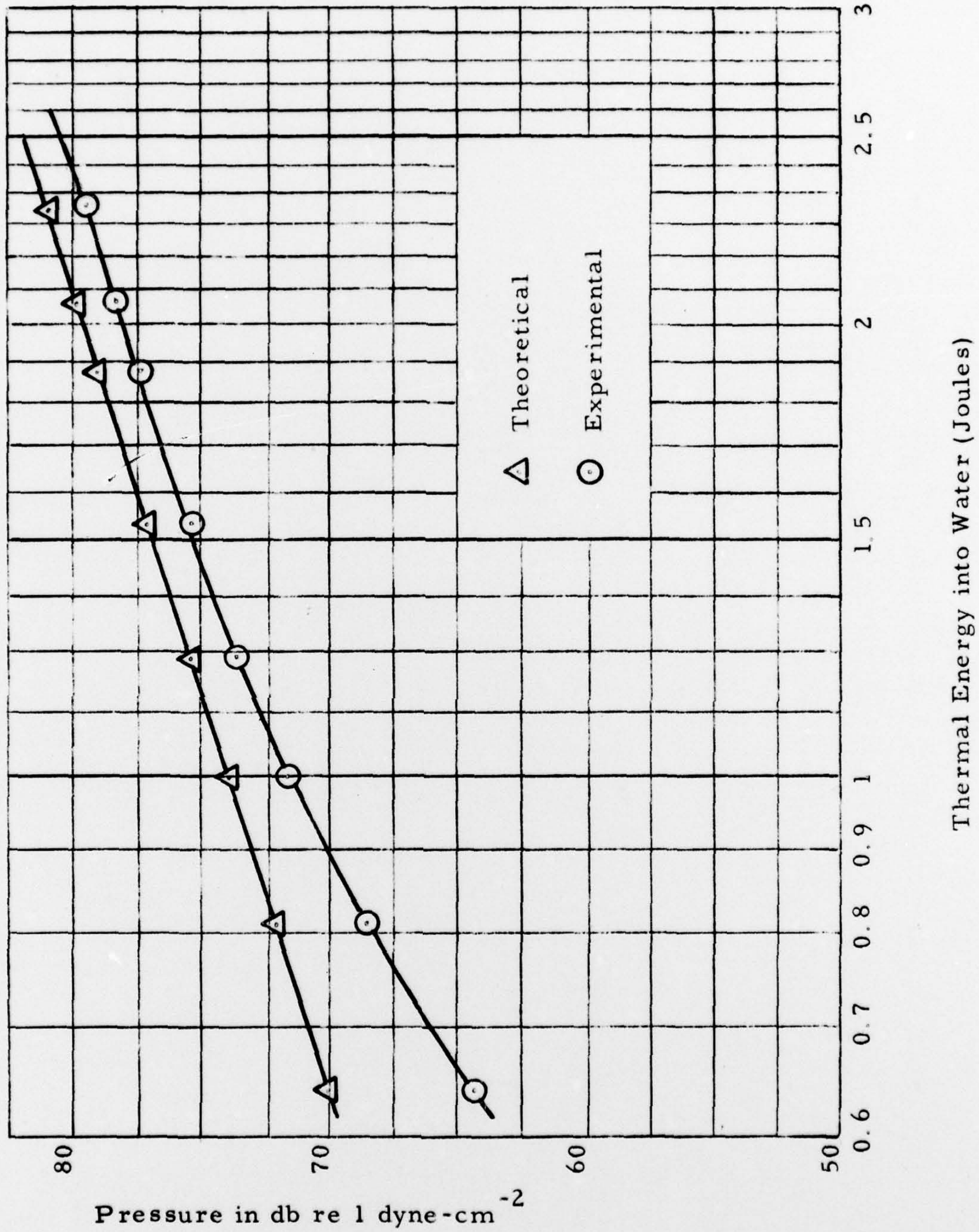
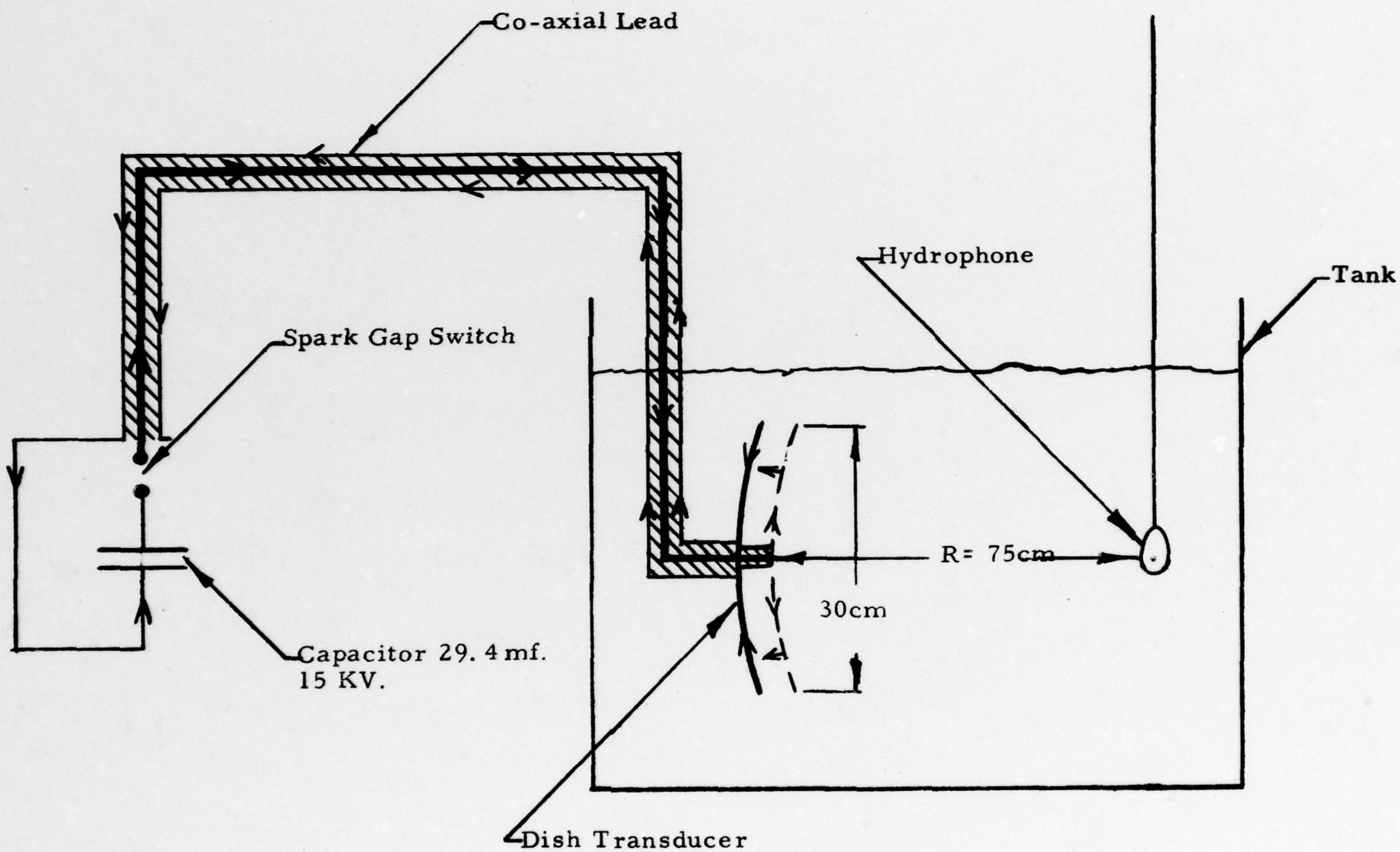


Figure 5. Comparison of Experimental and Theoretical Absolute Pressures at 65 cm. Distance from Projector

Figure 6 Schematic of Dish Transducer System

(Arrows Indicate Current Flow)



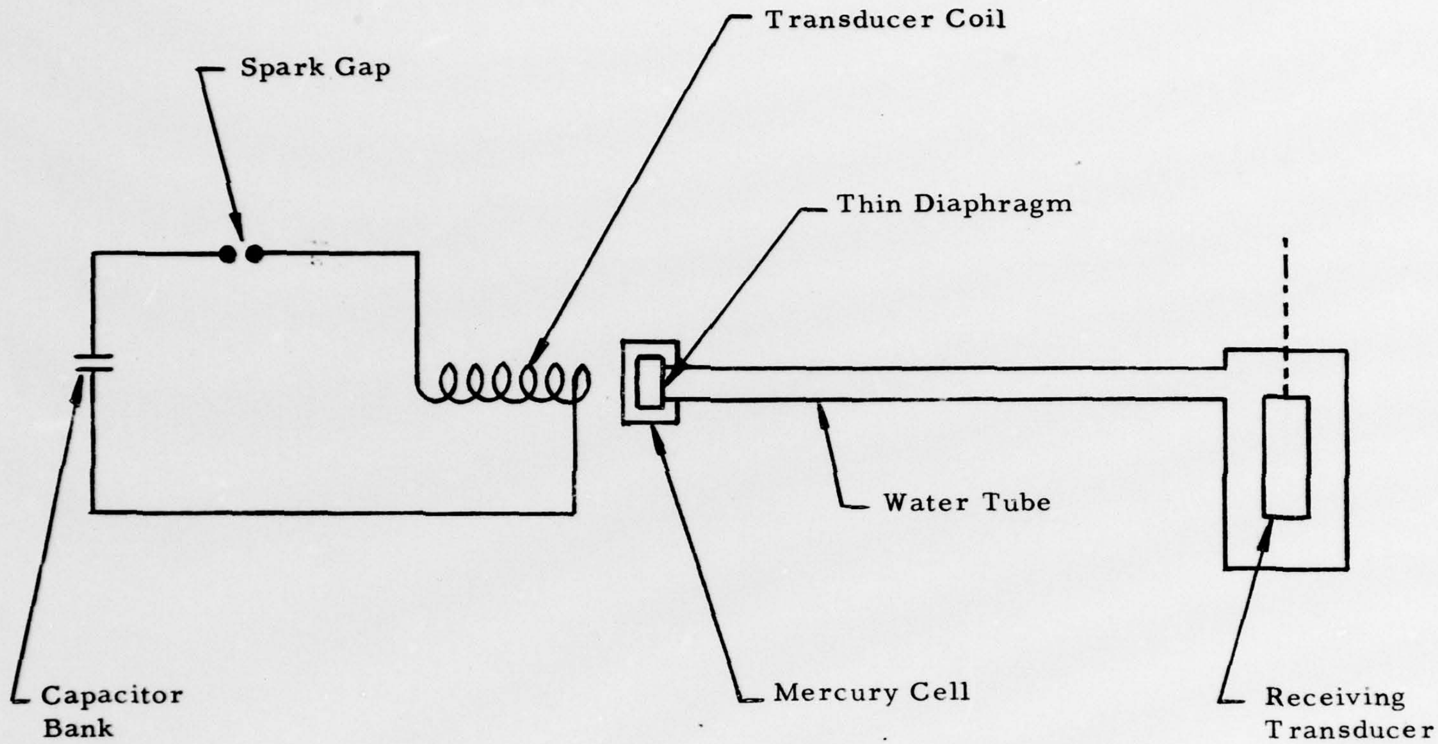
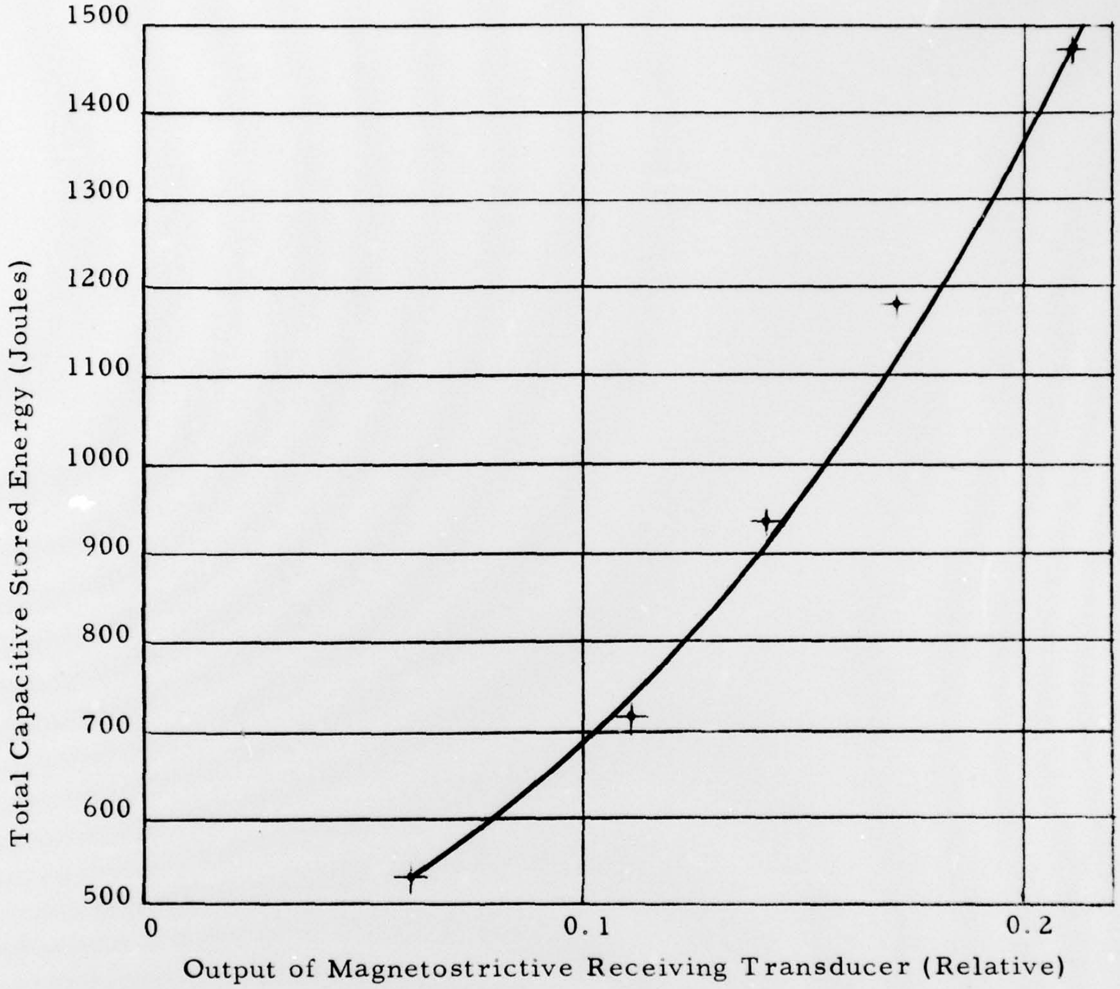


Figure 7. Experimental Arrangement for MHD Sound Generation in Mercury

Figure 8

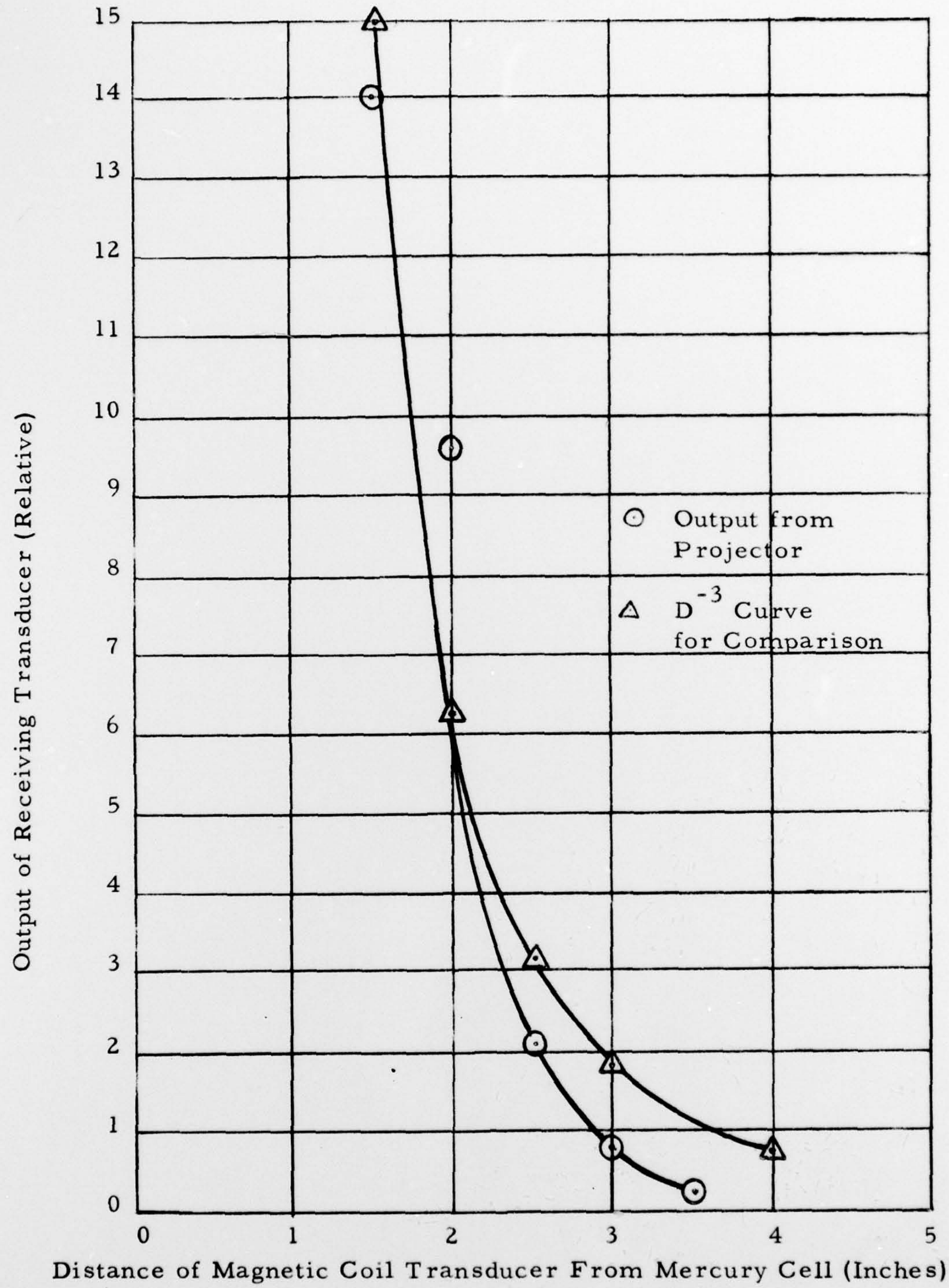
MHD Sound Generation in Mercury



Variation of Stored Energy Output of Receiving Transducer

Figure 9

MHD Sound Generation in Mercury



Variation of Output Voltage of Acoustic Transducer With Distance From Mercury Cell of Magnetic Coil Generating Transducer

II THEORETICAL STUDIES ON MAGNETOACOUSTICS

Introduction

Since the force which is responsible for magnetohydrodynamic sound is identifiable as the Lorentz force $\bar{j} \times \bar{B}$, ($\bar{j}$ current density, $\bar{B}$ = magnetic induction), it follows that an inseparable companion sound originates in joule heating ($j^2 r$, where r is the resistivity of the medium). Therefore, any discussion of magnetohydrodynamic effects is <u>incomplete</u> without a discussion of thermal effects. In the following paragraphs magnetohydrodynamic sound and thermal sound are analyzed according to the cited relationship.

A. Sound Generation by Magnetohydrodynamic Forces

The first order equation describing the propagation of pressure pulsations as generated by a body force F is the following:

$$\frac{1}{C_s^2}\frac{\partial^2 p}{\partial t^2} \quad \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} - \operatorname{div} \bar{F} \tag{1}$$

where p = pressure in dynes/cm^2

C_s = sound velocity $\frac{cm}{sec}$

$\bar{F}$ = dynes/cm^3 external force acting on the fluid

Equation (1) neglects, of course, considerations of shear and dilatational viscosity, and assumes also that pressure oscillations are isentropic.

When an electric current is caused to flow in a conducting fluid, magnetohydrodynamic effects are generated by the Lorentz force,

$$\bar{F} = \bar{j} \times \bar{B} \tag{2}$$

$\bar{j}$ is the vector electric current density in abs. amp/cm^2

$\bar{B}$ is the magnetic induction in gauss

Equation (1) can be solved directly as

$$p = -\frac{1}{4\pi} \int \frac{\operatorname{div}(\bar{j} \times \bar{B})}{D} \bigg|_{t - \frac{D}{c}} dx\, dy\, dz \tag{3}$$

where the integral is taken over all regions where div ($\bar{j}$ x $\bar{B}$) does not vanish.

The following vector operator identity is helpful in reducing equation (3) to more familiar terms:

$$\nabla \cdot (\bar{j} \times \bar{B}) = (\bar{\nabla} \times \bar{j}) \cdot \bar{B} - \bar{j} \cdot (\bar{\nabla} \times \bar{B}) \tag{4}$$

Maxwell's equations provide further reductions in terms of the equalities

$$\bar{\nabla} \cdot \bar{E} = -\frac{1}{c} \frac{\partial \bar{B}}{\partial t} \tag{5}$$

$$\bar{\nabla} \times \bar{H} = 4\pi \bar{J} + \frac{1}{c} \frac{\partial \bar{D}_1}{\partial t} \tag{6}$$

With these substitutions one finds, if $\frac{1}{c}\frac{\partial \bar{D}_1}{\partial t}$ is neglected (D_1 = displacement):

$$\bar{\nabla} \cdot (J \times \bar{E}) = -\frac{4\pi\sigma}{10^2}\left[\frac{\partial}{\partial t}\left(\frac{\mu H^2}{8\pi}\right) \cdot 10^{-7} + J^2 r\right] \tag{7}$$

where

σ = electrical conductivity Mho/cm

μ = magnetic permeability of the fluid

J = current density in practical amperes/cm^2

r = resistivity ohm-cm = $\frac{1}{\sigma}$

Accordingly, p becomes

$$p \doteq \frac{\sigma}{10^2} \int \left. \frac{\left[J^2 r + \frac{\partial}{\partial t}\left(\frac{\mu H^2}{8\pi}\right) 10^{-7}\right]}{D} \right|_{t - \frac{D}{c_s}} dx\,dy\,dz \tag{8}$$

If the wave lengths of the disturbance is great compared to the region over which the integration is to be carried out, the preceding formula can be simplified to

$$P = \frac{\mu_0}{\pi^2}\left[I_1 R + \frac{1}{I_1} \sum_{i,j} \frac{1}{2} L_{ij} I_i I_j \right] \cdot \frac{1}{D} \qquad (9)$$

where:

I_1 = total current in the liquid

i, j are electrical component designations, i = 1, j = 1, being the liquid

$\sum_{i,j} 1/2\ L_{ij} I_i I_j$ is the total magnetic field energy in the liquid

L_{ij} = self and mutual inductances in henries

R - resistance in ohms of the entire conducting region of the liquid

The actual distribution of electric current in a body of conducting fluid would depend on the physical description of the electrical system used to create the current and magnetic field. An obvious experimental complication is present in that any coil immersed in a fluid will, during an electrical discharge, experience self forces which will tend to move the coil turns with respect to each other. Such movement would provide a purely mechanical transducer action. Discrimination against mechanical effects of coil movements and thermal effects due to joule heating are essential to an experimental investigation of magnetohydrodynamic sound.

B. Sound Generation by Thermal Effects

The basic equation of motion of the liquid is given (Ref.(1)):

$$\rho \frac{\partial \bar{u}}{\partial t} = \rho \bar{F} - \rho(\bar{u}\cdot\bar{\nabla})\bar{u} - \bar{\nabla} p + (\eta' + 2\eta)\,\bar{\nabla}(\bar{\nabla}\cdot\bar{u}) - \eta\,\bar{\nabla}\times(\bar{\nabla}\times\bar{u})$$

$$+ \bar{\nabla}\cdot\bar{u}\,(\bar{\nabla}\eta') + 2(\bar{\nabla}\eta\cdot\bar{\nabla})\bar{u} + \bar{\nabla}\eta\times(\bar{\nabla}\times\bar{u}) \qquad (10)$$

where

ρ = mass density of the liquid

t = time (seconds)

$\bar{u}$ = particle velocity

$\bar{F}$ = vector body force per unit of mass

$\bar{\nabla}$ = gradient operator

p = total pressure

η' = dilatational viscosity coefficient

η = shear viscosity coefficient

$\bar{\nabla}\cdot(\ )$ = divergence operator

$\bar{\nabla}\times(\ )$ = curl operator

Ignoring effects of shear and dilatational viscosity and linearizing the remaining expressions in particle velocity we get

$$\rho \frac{\partial \bar{u}}{\partial t} = -\bar{\nabla} p + \rho \bar{F} \qquad (11)$$

Regarding $\bar{u}$ as a small quantity, the ρ multiplier can be taken as the unperturbed value, ρ_o. Further, in this case, we omit the body force $\bar{F}$. Theequation then simplifies to:

$$\rho_o \frac{\partial \bar{u}}{\partial t} = -\bar{\nabla} p \qquad (12)$$

Taking the divergence of both sides we find

$$\rho_0 \frac{\partial \nabla \cdot \bar{u}}{\partial t} = -\nabla^2 p \tag{13}$$

Since mass conservation in first order is

$$\frac{\partial \rho}{\partial t} = -\rho_0 \nabla \cdot \bar{u} \tag{14}$$

we have

$$\frac{\partial^2 \rho}{\partial t^2} = \nabla^2 p \tag{15}$$

Conservation of energy demands (Ref.):

$$T\,ds = -\frac{c_p}{\left(\frac{1}{V}\frac{\partial V}{\partial T}\right)_p}\frac{d\rho}{\rho} + \frac{c_p\,dp}{\rho\left(\frac{1}{V}\frac{\partial V}{\partial T}\right)_p\left(\frac{dp}{d\rho}\right)_s} \tag{16}$$

c_p = specific heat at constant pressure

$\left(\frac{1}{V}\frac{\partial V}{\partial T}\right)_p$ = thermal expansion coefficient at constant pressure

s = entropy per gram of fluid

If we write the preceding equation in terms of entropy per unit volume, s', we find

$$d\rho = \frac{1}{\left(\frac{dp}{d\rho}\right)_{s'}} dp - \left(\frac{1}{V}\frac{\partial V}{\partial T}\right)_p \frac{T\,ds'}{c_p} \tag{17}$$

$$Tds' = dQ, \text{ (heat added per unit volume)} \tag{18}$$

Linearizing and substituting into equation (6) we derive

$$\frac{1}{\left(\frac{dp}{d\rho}\right)_{s'}} \frac{\partial^2 p}{\partial t^2} - \frac{\left(\frac{1}{V}\frac{\partial V}{\partial T}\right)_p}{C_p} \frac{\partial^2 Q}{\partial t^2} = \nabla^2 p \tag{19}$$

For $\left(\frac{dp}{d\rho}\right)_{s'}$ we substitute c_s^2 the isentropic sound velocity. In the absence of conduction and radiation exchanges and viscous contributions to entropy, $\frac{\partial Q}{\partial t}$ is the joule heating rate, $J^2 r$ (J is the electrical current density amperes/cm^2, and r is the resistivity). The wave potential solution of equation (11) is then

$$p_{thermal} = \frac{\left(\frac{1}{V}\frac{\partial V}{\partial T}\right)_p}{4\pi C_p} \frac{\partial}{\partial t} \int \left.\frac{J^2 r}{D}\right|_{t-R} dx\,dy\,dz \tag{20}$$

where r is the resistivity of the fluid, ohm-cm, and J is the current density in practical amperes/cm^2, C_p is in joules/gm/deg at constant pressure. The result is identical with that given in the First Interim Technical Report, (Reference (4), Equations (27)-(32).

It should be noted that the working fluid can be heated by radiation, viscous heating, chemical action or particle fluxes such as neutron beams as well as by the discharge of electric current between electrodes. Each provides a means for generating sound when properly modulated.

REFERENCES

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